题：Show that the number of ways to colour n distinct balls in red and blue (not necessary all the balls to be coloured). In a way that the number of coloured (by red or blue) balls is odd is one less or one mor than the number of ways to colour them in a way that an even number of them are coloured (in red or blue). Find under what condition we have "less" or "more".

题：用红或蓝涂色 n 个不同的球（不必每个球都涂色）
证明 涂色奇数个球的方式少于或多于涂色偶数个球的方法
并求出 奇数方式少于或多于偶数的条件。

证明：涂色奇数球的方式为：$\binom{n}{1}2^1+\binom{n}{3}2^3+\binom{n}{5}2^5+\cdots=S_o$

涂色偶数的方式为：$\binom{n}{0}2^0+\binom{n}{2}2^2+\binom{n}{4}2^4+\cdots=S_e$

令 $f(x)=(1+x)^n=\sum_{k=0}^{n}\binom{n}{k}x^k=\binom{n}{0}+\binom{n}{1}x+\binom{n}{2}x^2+\cdots+\binom{n}{n}x^n$

$g(x)=(1-x)^n=\sum_{k=0}^{n}\binom{n}{k}(-x)^k=\binom{n}{0}-\binom{n}{1}x+\binom{n}{2}x^2+\cdots+(-1)^n\binom{n}{n}x^n$

$f(2)=3^n=\binom{n}{0}+\binom{n}{1}2+\binom{n}{2}2^2+\cdots+\binom{n}{n}2^n$

$g(2)=(-1)^n=\binom{n}{0}-\binom{n}{1}2+\binom{n}{2}2^2-\binom{n}{3}2^3+\cdots+(-1)^n\binom{n}{n}2^n$

$$S_o=\frac{f(2)-g(2)}{2}=\frac{3^n-(-1)^n}{2},\quad S_e=\frac{f(2)+g(2)}{2}=\frac{3^n+(-1)^n}{2}$$

when n is odd $S_o=\frac{3^n+1}{2}$, $S_e=\frac{3^n-1}{2}$, $S_o>S_e$

when n is even $S_o=\frac{3^n-1}{2}$, $S_e=\frac{3^n+1}{2}$, $S_o<S_e$