

題: $\int \frac{\cos x}{3\sin x + 5\cos x} dx = ?$

Solution: $I_c = \int \frac{\cos x}{3\sin x + 5\cos x} dx \xrightarrow{x = \frac{\pi}{2} - t} I_c = - \int \frac{\sin t}{3\cos t + 5\sin t} dt$

$$2I_c = \int \frac{\cos x}{3\sin x + 5\cos x} - \frac{\sin x}{3\cos x + 5\sin x} dx$$

$$= \int \frac{\cos x (3\cos x + 5\sin x) - \sin x (3\sin x + 5\cos x)}{(3\sin x + 5\cos x)(3\cos x + 5\sin x)} dx$$

$$= \int \frac{3(\cos^2 x - \sin^2 x)}{15 + 34\sin x \cos x} dx = 3 \int \frac{\cos 2x}{15 + 17\sin 2x} dx$$

$$= 3 \int \frac{\frac{1}{34}}{15 + 17\sin 2x} d(15 + 17\sin 2x)$$

$$= \frac{3}{34} \ln |15 + 17\sin 2x| + C$$

$$I_c = \frac{1}{2} \cdot \frac{3}{34} \ln |15 + 17\sin 2x| + C$$

$$= \frac{3}{68} \ln |15 + 17\sin 2x| + C$$