(1) 证明：$\binom{n}{1}-2\binom{n}{2}+3\binom{n}{3}-\cdots\cdots+(-1)^{n-1}n\binom{n}{n}=0$

证：∵ $k\binom{n}{k}=k\frac{n!}{(n-k)!\cdot k!}=n\frac{(n-1)!}{(n-k)!(k-1)!}=n\binom{n-1}{k-1}$

∴ $\binom{n}{1}-2\binom{n}{2}+3\binom{n}{3}-\cdots\cdots+(-1)^{n-1}n\binom{n}{n}$

$=n\binom{n-1}{0}-n\binom{n-1}{1}+n\binom{n-1}{2}-\cdots\cdots+(-1)^{n-1}n\binom{n-1}{n-1}$

$=n\left[\binom{n-1}{0}-\binom{n-1}{1}+\binom{n-1}{2}-\cdots\cdots+(-1)^{n-1}\binom{n-1}{n-1}\right]=n(1-1)^{n-1}$

$=0$

(2) 证明：$1+\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}+\cdots\cdots+\frac{1}{n+1}\binom{n}{n}=\frac{2^{n+1}-1}{n+1}$

证：∵ $\frac{1}{k+1}\binom{n}{k}=\frac{1}{k+1}\frac{n!}{(n-k)!k!}=\frac{1}{n+1}\frac{(n+1)!}{(n-k)!(k+1)!}=\frac{1}{n+1}\binom{n+1}{k+1}$

∴ $1+\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}+\cdots\cdots+n\binom{n}{n}$

$=1+\frac{1}{n+1}\binom{n+1}{2}+\frac{1}{n+1}\binom{n+1}{3}+\cdots\cdots+\frac{1}{n+1}\binom{n+1}{n+1}$

$=1+\frac{1}{n+1}\left[\binom{n+1}{2}+\binom{n+1}{3}+\cdots\binom{n+1}{n+1}\right]$

$=1+\frac{1}{n+1}\left[2^{n+1}-1-\binom{n+1}{1}\right]=1+\frac{2^{n+1}-n-2}{n+1}$

$=\frac{2^{n+1}-1}{n+1}$