

计算  $\iint_D (x^2 - y^2) e^{xy} dx dy$ . 其中  $D$  是由  $xy=1$ ,  $xy=4$  和  $y=x$ ,  $y=x+2$  围成的区域。

解: 二元坐标变换令  $u=xy$ ,  $v=x-y$   
则  $-2 \leq v \leq 0$ ,  $D = \{(u, v) \mid 1 \leq u \leq 4, -2 \leq v \leq 0\}$

$$\frac{\partial(u, v)}{\partial(x, y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} y & x \\ 1 & -1 \end{vmatrix} = -y - x = -(x+y)$$

$$J = \frac{\partial(x, y)}{\partial(u, v)} = \frac{1}{\frac{\partial(u, v)}{\partial(x, y)}} = -\frac{1}{x+y}$$

$$\iint_D (x^2 - y^2) e^{xy} dx dy = \iint_D (x-y)(x+y) e^{xy} \left| \frac{-1}{x+y} \right| du dv$$

$$= \iint_D v e^u du dv = \int_{-2}^0 \left( \int_1^4 v e^u du \right) dv$$

$$= \int_{-2}^0 v e^u \Big|_1^4 dv = (e^4 - e) \frac{v^2}{2} \Big|_{-2}^0$$

$$= -2(e^4 - e) = 2(e - e^4)$$