题关键词：矩阵特征值的特征向量的定义，矩阵特征值与其伴随阵和逆阵的关系。

题：Let A be a 3×3 matrix and its eigenvalues are $\lambda_1=1$, $\lambda_2=2$, $\lambda_3=3$,then

Find : (注：det(A)=|A|, adjA = A^*)

(1) det(A), (2) the eigenvalues of A^{-1},(3) the eigenvalues of adj(A), (4)the eigenvalues of $A^{-1}+A^2$

Solution:

(1) A is a 3×3 matrix and has 3 distinct eigenvalues the A is diagonalizable, then A is similar to diag((1,2,3)= $\begin{bmatrix}1 & & \\ & 2 & \\ & & 3\end{bmatrix}$, so det(A)=det(diag(1,2,3))=1X2X3=6

(2) Let none zero vector X is the eigenvector of A belonging to λ, so AX=λX =>（A-λI)X=0,

=> |A-λI|=0 => $|A^{-1}||A-\lambda I|=0$ => $|A^{-1}(A-\lambda I|=0$ => $|I-\lambda A^{-1}|=0$ => $|(1/\lambda)I-A^{-1}|=0$, so 1/λ is the eienvalue of A^{-1}, the eigenvalues of A^{-1} are 1, 1/2, 1/3.

(3) $A^{-1}=\frac{1}{\det(A)}adjA$, adjA=det(A)A^{-1}, let none zero vecto v is the eigenvector of adjA belonging to μ, so adjAv=μv => (adjA-μI)v=0 => |adjA-μI|=0 => $|\det(A)A^{-1}-\mu I|=0$ => $|\det(A)(A^{-1}-(\mu/\det(A))I)|=0$ => $(\det(A))^n|A^{-1}-(\mu/\det(A))I)|=0$ =>$|A^{-1}-(\mu/\det(A))I)|=0$, then μ/det(A) is the eigenvalue of A^{-1} , so 1/λ= μ/det(A) => μ=det(A)/λ, the eigenvaluves of adjA are 6,3,2.

(4) Let none zero vector X is the eigenvector of A belonging to λ, so AX=λX => AAX=AλX => $A^2X=\lambda AX=\lambda^2X$, so λ^2 is the eigenvalue of A^2, then the eigenvalues of A^2 are 1, 4, 9

A^{-1} is similar to diag(1,1/2,1/3) and A^2 is similar to diag(1,4,9), so $A^{-1}+A^2$ is similar to diag(1,1/2,1/3)+diag(1,4,9)=diag(2,9/2,28/3), then the eigenvalues of $A^{-1}+A^2$ are 2, 9/2 , 28/3 .